

Super Flavorsymmetry with Multiple Higgs Doublets^{*}

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Abstract

The meaning of non-abelian super flavorsymmetry in the presence of Multiple Higgs Doublets is twofold: The first one is to order the Yukawa sector of the standard model, while generating the Cheng-Sher suppression of tree-level Flavor Changing Neutral Currents (FCNCs), and the second one is to soften the fine tuning problem in the scalar potential and at the same time to suppress FCNCs coming from the soft breaking of supersymmetry. If spontaneous CP violation is combined with non-abelian super flavorsymmetry the CP phases in the soft supersymmetry breaking sector can disappear. This feature of super flavorsymmetry is reviewed and elaborated in a supersymmetric model of flavor based on the finite group Q_6 . The predictions of the model in fermion mass and mixing, and flavor and CP violation can be tested in future experiments.

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I. INTRODUCTION

Introducing flavor symmetry based on a non-abelian finite group into the standard model (SM) means going beyond the SM [1]. The main reason of the introduction of flavor symmetry lies in the Yukawa sector of the SM, because the most of the free parameters of the SM are involved in this sector, and there exists within the SM no principle how to fix its structure. There are many guidelines to go beyond the SM. Therefore, depending on the guideline, how to introduce non-abelian flavor symmetry is different [2]-[17]. The difference of the guidelines may be summarized as:

1. How it is broken.
Since there exists no exact non-abelian flavor symmetry in the SM, it has to be broken: It is spontaneously, softly or hardly broken.
2. The scale of its breaking.
It is a low energy scale or a much higher scale, e.g., a GUT scale.
3. Renormalizability.
The extended model is renormalizable or non-renormalizable.
4. On which sector it is effectively acting: On the quark sector; on the lepton sector; or on both the quark and the lepton sector.

In addition to the above four categories we can add whether the model is supersymmetric or not, it is a four dimensional model or not, and whether CP is spontaneously, softly or explicitly broken. Note that no gauge group and no flavor group are fixed yet. However, except for the existence of dark matter and the neutrino mixing and masses, despite the great efforts of experimentalists, there is no experimental hint how to go beyond the SM.

Under these circumstances I prefer to attach importance to the experimental verifiability of a model. It is clear that the lower the scale of the breaking of flavor symmetry is, the more verifiable is the model. Moreover, if flavor symmetry is hardly broken, it is not testable in a strict sense. Further, to decide whether the model should be renormalizable or not, I would like to follow the attempt of Kobayashi and Maskawa [18], who introduced CP violation within the *renormalizable extension* of the two generation SM. From these considerations I decided to restrict myself, in this contribution to the special issue, to renormalizable models of flavor, where non-abelian discrete flavor symmetries are at most softly broken at low energy. One immediately realizes that it is impossible to present a realistic model along this line, unless one introduces multiple Higgs doublets which belong to non-trivial representations of the corresponding non-abelian finite group. Therefore, flavor symmetry is at least spontaneously broken at the electroweak scale together with the electroweak gauge symmetry. This can lead to severe problems with Flavor Changing Neutral Currents (FCNCs), if the extra Higgs bosons are not heavy enough [19]. I will not discuss the details of the different constraints such as $\mu \rightarrow 3e$ coming from flavor violations (see [20] for instance). I simply assume that their masses are larger than several TeV and the electroweak precision constraints are satisfied [21].

In contrast to two Higgs doublet models without non-abelian flavor symmetry [22], the size of the Yukawa couplings are not arbitrary, because fermion masses and mixing would not come out right. However, the scale is not $O(100)$ TeV [19], because the Cheng-Sher mechanism [23], which suppresses the tree-level FCNCs, is partially working thanks to flavor

symmetry and making the scale of flavor symmetry decrease down to tera scale. The presence of extra heavy Higgs doublet bosons of tera scale causes another problem, a fine tuning in the scalar potential. The fine tuning can be made stable against radiative corrections by supersymmetry [24–26]. This is why I will concentrate on supersymmetric models of flavor. Due to the very nature of non-renormalization theorem, extended to softly broken supersymmetric theories [27, 28], the soft breaking terms of flavor symmetry can be classified, in addition to according to their canonical dimensions (Symanzik’s theorem [29]). I will discuss this matter in a general manner in Sect. 2 and 3.

The meaning of non-abelian flavor symmetry in supersymmetric models is twofold: The first one is to introduce a principle in the Yukawa sector as it is the case for non-supersymmetric models, while generating the Cheng-Sher suppression, and the second one is to suppress FCNCs coming from the soft breaking of supersymmetry [30]–[35], while supersymmetry softens the fine tuning problem in the scalar potential. This mechanism to suppress FCNCs is alternative to hidden sector supersymmetry breaking. (The both mechanisms can be of course combined.) The suppression mechanism has been observed in many concrete models [7, 11, 36]–[46], and it is in fact very general. In certain models, like the one [7] which I will discuss in detail, the CP violating phases in the soft supersymmetry breaking (SSB) sector disappear, if CP is spontaneously broken: A phase self-alignment works [7].

From Sect. 4 on I will discuss the specific supersymmetric flavor model based on the finite flavor group Q_6 [7, 11], [43]–[46]. This model will illustrate the basic features of any realistic supersymmetric flavor models with multiple Higgs doublet supermultiplets [47]. The predictions on fermion mass and mixing are discussed in Sect. 5, and I will show in Sect. 6 how FCNCs and CP violations coming from the supersymmetry breaking are suppressed by Q_6 flavor symmetry. Sect. 7 is devoted to Summary.

II. FLAVOR SYMMETRY AND NON-RENORMALIZATION THEOREM IN SOFTLY BROKEN SUPERSYMMETRIC THEORIES

Flavor symmetry can control the structure of the independent parameters of a theory. In supersymmetric theories, moreover, the non-renormalization theorem allows to drop certain couplings and also to relate them with each other, without facing contradictions with renormalizability. We would like to discuss the role of the non-renormalization theorem for flavor symmetry and vice versa. We begin by considering a generic $N = 1$ supersymmetric gauge theory whose superpotential is given by

$$W(\Phi) = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j. \quad (1)$$

The soft supersymmetry breaking (SSB) Lagrangian is given by

$$L(\Phi, W) = - \left(\int d^2\theta \eta \left(\frac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}b^{ij}\Phi_i\Phi_j + \frac{1}{2}M_g W_A^\alpha W_{A\alpha} \right) + \text{h.c.} \right) \\ - \int d^4\theta \tilde{\eta} \eta \overline{\Phi^j} (m^2)_j^i (e^{2gV})_i^k \Phi_k, \quad (2)$$

where $\eta = \theta^2$, $\tilde{\eta} = \tilde{\theta}^2$ are the external spurion superfields and M_g is the gaugino mass.

To proceed we take into account the presence of flavor symmetry. We recall that the D-terms are renormalized and the wave function renormalization can mix matter superfields Φ_i

in general. If a non-diagonal (infinite) kinetic term is induced, a corresponding non-diagonal counter term should be added. In what follows we assume that the kinetic mixing among Φ_i 's is forbidden by flavor symmetry. This implies that the anomalous dimensions γ_j^i are diagonal, i.e., $\gamma_j^i = \delta_j^i \gamma_j$. Then the β functions of Y, μ, h and m^2 are given by [28, 48]

$$\beta_Y^{ijk} = Y^{ijk}(\gamma_i + \gamma_j + \gamma_k), \quad \beta_\mu^{ij} = \mu^{ij}(\gamma_i + \gamma_j), \quad (3)$$

$$\beta_h^{ijk} = (h^{ijk} - 2Y^{ijk}\mathcal{O})(\gamma_i + \gamma_j + \gamma_k), \quad \beta_b^{ij} = (b^{ij} - 2\mu^{ij}\mathcal{O})(\gamma_i + \gamma_j), \quad (4)$$

$$(\beta_{m^2})_l = \left[\Delta + X \frac{\partial}{\partial g} \right] \gamma_l, \quad \mathcal{O} = \left(M_g g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y^{lmn}} \right), \quad (5)$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M_g|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn} \frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn} \frac{\partial}{\partial Y^{lmn}}, \quad (6)$$

where $(\gamma_1)_i = \mathcal{O}\gamma_i$, $Y_{lmn} = (Y^{lmn})^*$, and $\tilde{Y}^{ijk} = Y^{ijk}(m_i^2 + m_j^2 + m_k^2)$, $X = \frac{-|M_g|^2 C(G) + \sum_l m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}$, and $T(R_l)$ is the Dynkin index of R_l , and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G . From these equations we may conclude (which is basically Symanzik's theorem [29] applied to softly broken supersymmetric gauge theories):

Th1. The μ sector can have a flavor symmetry which is different from the flavor symmetry of the Yukawa sector if both symmetries are compatible with respect to renormalization of μ^{ij} . That is, if $\mu^{ij}(\gamma_i + \gamma_j) \sim \mu^{ij}$.

Th2. It is consistent to introduce into the soft tri-linear couplings the same flavor symmetry as that of the Yukawa couplings, even if it is violated in other sectors.

Th3. The flavor symmetry which protects the kinetic mixing among Φ_i 's ensures that $(m^2)_j^i$ are diagonal. If the Yukawa couplings and soft tri-linear couplings have the flavor symmetry, the soft scalar mass terms, too, can have the flavor symmetry, even if the μ and b terms do not respect the flavor symmetry.

Th4. The b terms associated with the μ terms should always exist (see (5)). But the b sector has no influence on the infinite renormalization of the parameters in other sectors. So the violation of a symmetry in the b sector is doubly soft. Similarly, the soft scalar mass terms and soft tri-linear couplings have no influence on the infinite renormalization of the b terms.

III. SOFTLY BROKEN FLAVOR SYMMETRIES AND SUPERSYMMETRY

We restrict ourselves to the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and require renormalizability as it is announced. As in the non-supersymmetric case, realistic softly broken supersymmetric models with a low energy flavor symmetry will contain certain number of Higgs doublet supermultiplets pairs Φ_I^u and Φ_I^d . The superpotential in the matter sector is given by $W_Q + W_L + W_M$, where

$$W_Q = Y_{ij}^{uI} Q_i U_j^c \Phi_I^u + Y_{ij}^{dI} Q_i D_j^c \Phi_I^d, \quad W_L = Y_{ij}^{eI} L_i E_j^c \Phi_I^d + Y_{ij}^{\nu I} L_i N_j^c \Phi_I^u, \quad (7)$$

$$W_M = \frac{1}{2} M_{ij} N_i^c N_j^c, \quad (8)$$

where Q and L stand for $SU(2)_L$ doublets of the quark and lepton supermultiplets, respectively, where i and j are flavor group indices. Similarly, U^c, E^c and N^c stand for $SU(2)_L$

singlets of the quark, charged lepton and right-handed neutrino supermultiplets. In the following discussions we denote the scalar and fermionic components of the supermultiplets as

$$Q = (\tilde{q}_L , q_L) = \begin{pmatrix} \tilde{u}_L , u_L \\ \tilde{d}_L , d_L \end{pmatrix} , \quad U^c = (\tilde{u}_R^* , u_R^c) , \quad (9)$$

and similarly for D^c, L , etc. The Higgs supermultiplets are

$$\Phi^u = (h^u , \tilde{h}^u) = \begin{pmatrix} h^{u+} , \tilde{h}^{u+} \\ h^{u0} , \tilde{h}^{u0} \end{pmatrix} , \quad \Phi^d = (h^d , \tilde{h}^d) = \begin{pmatrix} h^{d0} , \tilde{h}^{d0} \\ h^{d-} , \tilde{h}^{d-} \end{pmatrix} , \quad (10)$$

and we assume that the neutral components of the Higgs fields, h^{u0} and h^{d0} , acquire a (complex) VEV.

The μ terms: The most general, renormalizable, Higgs superpotential W_H has the form

$$W_H = \sum_{I,J} \mu_{IJ} \Phi_I^u \Phi_J^d . \quad (11)$$

The superpotential W_H often has a symmetry larger, quite often continuous symmetry, than the one in the Yukawa sector. The large continuous symmetry has to be explicitly broken in the scalar potential to avoid the appearance of the massless NG bosons and fermions. Another problem is that certain fermionic components of Φ_I are massless. The most general form of the mass matrix \mathbf{M}_N^F of the neutralinos in the presence of N pairs of the Higgs doublet supermultiplets can be written as

$$\mathbf{M}_N^F = \begin{pmatrix} \mathbf{M}_{gg} & \mathbf{M}_{g1} & \mathbf{M}_{g2} & \mathbf{M}_{g3} & \cdots & \mathbf{M}_{gN} \\ \mathbf{M}_{g1}^T & \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} & \cdots & \mathbf{M}_{1N} \\ \mathbf{M}_{g2}^T & \mathbf{M}_{12}^T & \mathbf{M}_{22} & \mathbf{M}_{23} & \cdots & \mathbf{M}_{2N} \\ \mathbf{M}_{g3}^T & \mathbf{M}_{13}^T & \mathbf{M}_{23}^T & \mathbf{M}_{33} & \cdots & \mathbf{M}_{3N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_{gN}^T & \cdots & \cdots & \cdots & \cdots & \mathbf{M}_{NN} \end{pmatrix} , \quad (12)$$

where

$$\mathbf{M}_{gg} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} , \quad \mathbf{M}_{gI} = \begin{pmatrix} s_W(v_I^u/v)M_Z & -s_W(v_I^d/v)M_Z \\ -c_W(v_I^u/v)M_Z & c_W(v_I^u/v)M_Z \end{pmatrix} , \quad \mathbf{M}_{IJ} = \begin{pmatrix} 0 & -\mu_{IJ} \\ -\mu_{IJ} & 0 \end{pmatrix} , \quad (13)$$

$s(c)_W = \sin(\cos)\theta_W$, and $M_{1,2}$ are gaugino masses. Similarly, the mass matrix of the charginos \mathbf{M}_C has the form

$$\mathbf{M}_C^F = \begin{pmatrix} M_2 & \cdots & (v_J^d/v)M_W & \cdots \\ \vdots & & \vdots & \\ (v_I^u/v)M_W & \cdots & \mu_{IJ} & \cdots \\ \vdots & & \vdots & \end{pmatrix} . \quad (14)$$

To give the massless fermions a mass we can break flavor symmetry at the superpotential level, which however should be consistent with **Th1**.

The soft scalar mass terms: Since flavor symmetry for the soft scalar mass terms of the matter fields plays a crucial role to suppress FCNCs in the SSB sector, we assume that

flavor symmetry is not broken by the soft scalar mass terms. This requires implicitly that flavor symmetry is respected by the soft tri-linear couplings, because the breaking by the soft tri-linear couplings is not consistent with the assumption that flavor symmetry is respected by the soft scalar mass terms. The most general form of the soft scalar mass terms for the multi Higgs doublet fields is

$$\mathcal{L}_{m_H^2} = \sum_I \left(m_{H_I^u}^2 (h_I^u)^* h_I^u + m_{H_I^d}^2 (h_I^d)^* h_I^d \right) + \sum_{I < J} \left(m_{H_{IJ}^u}^2 (h_I^u)^* h_J^u + m_{H_{IJ}^d}^2 (h_I^d)^* h_J^d + h.c. \right) . \quad (15)$$

The superpotential (7) gives rise to quadratic terms, too:

$$\mathcal{L}_{\mu^2} = \sum_{I,J,K} \left(\mu_{IK}^* \mu_{JK} (h_I^u)^* h_J^u + \mu_{KI}^* \mu_{KJ} (h_I^d)^* h_J^d \right) . \quad (16)$$

The b terms:

The b sector should contain at least terms which correspond to the μ terms in W_H :

$$\mathcal{L}_b = \sum_{IJ} \left(b_{IJ} h_I^u h_J^d + h.c. \right) . \quad (17)$$

If flavor symmetry should be exact, then b_{IJ} in (11) have the same structure as μ_{IJ} in (11). As we discussed, the breaking of flavor symmetry by the b terms is doubly soft; it is soft, because the canonical dimension of the breaking operators is two, and the breaking has no influence on the infinite renormalization of the soft scalar masses (see **Th2**). Therefore, if the Higgs superpotential W_H given in (11) has an accidental continuous symmetry, one can break it softly by the b terms to avoid the appearance of the NG bosons and fermions.

If the b parameters are complex, CP is explicitly broken by the b terms; the breaking is doubly soft. CP breaking by the b terms is an economic way, because one can overcome the domain wall problem which might appear when discrete flavor symmetries are spontaneously broken, and one may moreover obtain extra CP phases to make electroweak baryogenesis possible.

Softly broken CP invariance: In the presence of Higgs doublet pairs only, the quartic terms in the Higgs scalar potential come only from the D-terms, which are positive semi definite. Therefore, to show that the potential minimum is not at the origin, one needs to consider only the quadratic terms

$$\mathcal{L}_{m_H^2} + \mathcal{L}_b = \frac{1}{2} \sum_{I,J} \begin{pmatrix} \varphi_I^u \\ \chi_I^u \\ \varphi_I^d \\ \chi_I^d \end{pmatrix}^T (\mathbf{M}_H^2)_{IJ} \begin{pmatrix} \varphi_J^u \\ \chi_J^u \\ \varphi_J^d \\ \chi_J^d \end{pmatrix} , \quad (18)$$

where $h_I^{u,d} = (\varphi_I^{u,d} + i\chi_I^{u,d})/\sqrt{2}$, and the 4×4 matrix $(\mathbf{M}_H^2)_{IJ}$ has the form

$$(\mathbf{M}_H^2)_{IJ} = \begin{pmatrix} m_{H_{IJ}^u}^2 + \mu_{IK}^* \mu_{JK} & m_{H_{IJ}^u}^2 + \mu_{IK}^* \mu_{JK} & \text{Re}(b_{IJ}) & -\text{Im}(b_{IJ}) \\ m_{H_{IJ}^u}^2 + \mu_{IK}^* \mu_{JK} & m_{H_{IJ}^u}^2 + \mu_{IK}^* \mu_{JK} & -\text{Im}(b_{IJ}) & -\text{Re}(b_{IJ}) \\ \text{Re}(b_{JI}) & -\text{Im}(b_{JI}) & m_{H_{IJ}^d}^2 + \mu_{KI}^* \mu_{KJ} & m_{H_{IJ}^d}^2 + \mu_{KI}^* \mu_{KJ} \\ -\text{Im}(b_{JI}) & -\text{Re}(b_{JI}) & m_{H_{IJ}^d}^2 + \mu_{KI}^* \mu_{KJ} & m_{H_{IJ}^d}^2 + \mu_{KI}^* \mu_{KJ} \end{pmatrix} . \quad (19)$$

The relevant matrix is the $4N \times 4N$ matrix \mathcal{M}_H^2 (N is the number of the Higgs doublet pairs)

$$\mathcal{M}_H^2 = \begin{pmatrix} (\mathbf{M}_H^2)_{11} & (\mathbf{M}_H^2)_{11} & \dots \\ (\mathbf{M}_H^2)_{21} & (\mathbf{M}_H^2)_{22} & \dots \\ \dots & \dots & \dots \end{pmatrix}, \quad (20)$$

whose eigenvalues control the behavior of the Higgs scalar potential in the $4N$ dimensional space. We find that all the eigenvalues of \mathcal{M}_H^2 are doubly degenerate, and that two orthogonal eigenvectors of the same eigenvalue can be always written in the form $\vec{u}_A = (u_1, u_2, u_3, u_4, \dots)$ and $\vec{u}_B = (u_2, -u_1, -u_4, u_3, \dots)$. This is due to the $U(1)_Y$ gauge invariance: All the directions defined by a linear combination of \vec{u}_A and \vec{u}_B are physically equivalent. That is, one can make an appropriate linear combination such that at least one imaginary part vanishes. If on the other hand all the imaginary parts of m_H^2 's and b 's vanish, then the eigenvectors can always be written as $(c_\theta u_1, -s_\theta u_1, c_\theta u_3, s_\theta u_3, c_\theta u_5, -s_\theta u_5, \dots)$, where $c_\theta = \cos \theta$ and $s_\theta = \sin \theta$, and the angle θ is, thanks to $U(1)_Y$, a free-choosable parameter. Therefore, we can always choose θ to be zero, i.e., all the imaginary parts vanish in this direction, implying that spontaneous CP violation at the tree level is not possible [49]. Therefore, at least some of m_H^2 or/and of b should be complex so that the Higgs doublet fields can acquire imaginary VEV. Needless to say that negative eigenvalues of (20) have to exist for the potential minimum to be located other than at the origin.

The tri-linear couplings : The soft tri-linear couplings corresponding to the superpotential (7) is

$$\mathcal{L}_A = h_{ij}^{uI} \tilde{q}_{Li} \tilde{u}_{Rj}^* h_I^u + h_{ij}^{dI} \tilde{q}_{Li} \tilde{u}_{Rj}^* h_I^d + h_{ij}^{eI} \tilde{l}_{Li} \tilde{e}_{Rj}^* h_I^d + h_{ij}^{\nu I} \tilde{l}_{Li} \tilde{\nu}_{Rj}^* h_I^u + h.c. . \quad (21)$$

According to **Th2** and **Th3**, the soft tri-linear couplings and soft scalar mass terms can have the same flavor symmetry as that of the Yukawa sector, even if flavor symmetry is softly broken in the μ sector and by the b terms. Further, the superpotential (7) gives rise to tri-linear couplings, too:

$$\mathcal{L}_\mu = \mu_{IJ} (Y_{ij}^{dJ})^* \tilde{Q}_i^\dagger \tilde{d}_{Rj} h_I^u - \mu_{IJ} (Y_{ij}^{uI})^* \tilde{Q}_i^\dagger \tilde{u}_{Rj} h_J^d + \mu_{IJ} (Y_{ij}^{eJ})^* \tilde{L}_i^\dagger \tilde{e}_{Rj} h_I^u - \mu_{IJ} (Y_{ij}^{\nu I})^* \tilde{L}_i^\dagger \tilde{\nu}_{Rj} h_J^d + h.c. , \quad (22)$$

which contributes to the soft left-right mass matrices of the sfermions as we will see later on.

IV. A CONCRETE MODEL OF FLAVOR BASED ON THE FINITE GROUP Q_6 [7]

In Table 1 the Q_6 assignment of the quark, lepton and Higgs chiral supermultiplets is given ¹. In addition to Q_6 flavor symmetry, a flavor universal Z_4 symmetry is introduced. Owing to this Z_4 , even after spontaneous symmetry breaking, unbroken interchange symmetries $\mathcal{P}_{I,II}$ survive in the Higgs potential [46]. These symmetries $\mathcal{P}_{I,II}$ along with Q_6 enable us to reduce significantly the number of parameters in the fermion mass matrices. This reduction

¹ The finite group Q_6 was also considered by Frampton and Kong in [3]. See [7] for the tensor product. The same model exists for Q_{2N} if N is odd and a multiple of 3.

of parameters leads to a sum rule involving quark masses and mixing parameters [7]. We assume that CP violation is spontaneously broken, which is perhaps more satisfying than the usual assumption of explicit CP violation. Nevertheless, the dominant source of CP violation in the quark sector is the Kobayashi-Maskawa mechanism. With the spontaneous breaking of CP, the CP problem that generically exists in the SSB sector can be softened in a rather simple way [7, 11, 41, 44, 46].

TABLE I: Particle content of the Q_6 model along with their transformation rule under $Q_6 \times Z_4$.

	Q	L	Q_3	L_3	U^c, D^c, N^c, E^c	U_3^c, D_3^c	N_3^c	E_3^c	$\Phi^{u,d}$	$\Phi_3^{u,d}$	S	S_3	T	T_3	U
Q_6	2	2'	1'	1	2'	1'''	1''	1	2'	1'''	2	1	2'	1'	1
Z_4	$-i$	$-i$	$-i$	$-i$	1	1	1	1	i	i	-1	-1	+	+	+
R	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+

A. The Higgs sector

A certain set of SM singlet Higgs fields is needed to break Q_6 flavor symmetry spontaneously while avoiding pseudo NG bosons [46]. The minimal such set involves **2**, **2'**, **1'** and two **1**'s of Q_6 . The SM singlet S 's mix the Q_6 doublets $\Phi^{u,d}$ with the Q_6 singlets $\Phi_3^{u,d}$. An accidental $O(2)$ symmetry, which would exist without the Q_6 doublet T in the Higgs potential, is violated by the cubic coupling of T . The T_3 is needed for the Majorana mass of N_3^c , and the Q_6 singlet U is introduced to generate spontaneous CP violation and to induce spontaneous symmetry breaking of $Q_6 \times Z_4$ within the SM singlet sector.

The most general Higgs superpotential involving the Higgs multiplets invariant under the $Q_6 \times Z_4$ symmetry along with the matter parity has the form $W_{\text{Higgs}} = W_U + W_{ST} + W_H$, where

$$W_U = \mu_U U^2 + \lambda U^3 + (\lambda_1 S_3^2 + \lambda_2 T_3^2 + \lambda_3 T \cdot T) U, \quad (23)$$

$$W_{ST} = \mu_{S_3} S_3^2 + \mu_T T \cdot T + \mu_{T_3} T_3^2 + \lambda'_3 T \cdot (T \otimes T) \\ + \lambda'_1 [-2S_2 S_1 T_1 + (S_1^2 - S_2^2) T_2] + \lambda'_2 S \cdot S T_3, \quad (24)$$

$$W_H = \lambda''_1 \Phi_3^u (\Phi^d * S) + \lambda''_2 (\Phi^u * S) \Phi_3^d + \lambda''_3 (\Phi^u \cdot \Phi^d) S_3 \quad (25)$$

with $x \cdot y = x_1 y_1 + x_2 y_2$, $x * y = x_1 y_2 + x_2 y_1$, $x \star y = x_1 y_2 - x_2 y_1$, $x \cdot (y \otimes z) = x_1 (-y_1 z_1 + y_2 z_2) + x_2 (y_1 z_2 + y_2 z_1)$. The Higgs potential contains F -terms derived from Eqs. (23)- (25), D -terms associated with $SU(2)_L \times U(1)_Y$ gauge symmetry, and the following SSB Lagrangian ²

$$\mathcal{L}_{\text{soft}} = m_U^2 |U|^2 + m_S^2 (|S_1|^2 + |S_2|^2) + m_{S_3}^2 |S_3|^2 + m_T^2 (|T_1|^2 + |T_2|^2) + m_{T_3}^2 |T_3|^2 \\ + m_{h_3^u}^2 |h_3^u|^2 + m_{h_3^d}^2 |h_3^d|^2 + m_{h^u}^2 (|h_1^u|^2 + |h_2^u|^2) + m_{h^d}^2 (|h_1^d|^2 + |h_2^d|^2) \\ + \{ B_U U^2 + B_{S_3} S_3^2 + B_T T \cdot T + B_{T_3} T_3^2$$

² We use the same symbol for the scalar components as the superfields for T, S and U .

$$\begin{aligned}
& + [A U^2 + A_1 S_3^2 + A_2 T_3^2 + A_3 (T \cdot T)] U + A'_3 T \cdot (T \otimes T) \\
& + A'_1 [-2S_2 S_1 T_1 + (S_1^2 - S_2^2) T_2] + A'_2 S \cdot S T_3 \\
& + A''_1 h_3^u (h^d * S) + A''_2 (h^u * S) h_3^d + A''_3 (h^u \cdot h^d) S_3 + h.c. \} .
\end{aligned} \tag{26}$$

We assume CP invariance, which implies that all the Yukawa couplings and the parameters in the Higgs potential are real. The Higgs potential derived from (23) -(25), and (26) including the D -terms admits two interesting minima which leave two separate discrete symmetries \mathcal{P}_I and \mathcal{P}_{II} unbroken:

$$\begin{aligned}
\mathcal{P}_I : h_1^u \leftrightarrow h_2^u, h_1^d \leftrightarrow h_2^d, S_1 \leftrightarrow S_2, T_2 \rightarrow -T_2, \\
h_3^u \rightarrow h_3^u, h_3^d \rightarrow h_3^d, S_3 \rightarrow S_3, T_1 \rightarrow T_1, T_3 \rightarrow T_3, U \rightarrow U,
\end{aligned} \tag{27}$$

$$\begin{aligned}
\mathcal{P}_{II} : h_1^u \leftrightarrow h_2^{u*}, h_1^d \leftrightarrow h_2^{d*}, S_1 \leftrightarrow S_2^*, T_2 \rightarrow -T_2^*, \\
h_3^u \rightarrow h_3^{u*}, h_3^d \rightarrow h_3^{d*}, S_3 \rightarrow S_3^*, T_1 \rightarrow T_1^*, T_3 \rightarrow T_3^*, U \rightarrow U^*.
\end{aligned} \tag{28}$$

The VEVs of the various Higgs fields can be consistently chosen such that these symmetries remains unbroken. We have explicitly found local minima at which CP and $Q_6 \times Z_4$ are spontaneously broken. We however will not present the full analysis of the potential. Furthermore, in the following discussions of the paper we consider only the second case \mathcal{P}_{II} . The \mathcal{P}_I invariant case works in a similar way.

The \mathcal{P}_{II} invariance enables us to choose a ground state given by

$$\begin{aligned}
\langle h_1^u \rangle &= v_1^u e^{-i\phi^u}, \quad \langle h_2^u \rangle = v_1^u e^{i\phi^u}, \quad \langle h_1^d \rangle = v_1^d e^{-i\phi^d}, \quad \langle h_2^d \rangle = v_1^d e^{i\phi^d}, \\
\langle h_3^u \rangle &= v_3^u, \quad \langle h_3^d \rangle = v_3^d, \quad \langle S_1 \rangle = v_S e^{-i\phi_S}, \quad \langle S_2 \rangle = v_S e^{i\phi_S}, \quad \langle S_3 \rangle = v_{S_3}, \\
\langle T_1 \rangle &= v_{T_1}, \quad \langle T_2 \rangle = -iv_{T_2}, \quad \langle T_3 \rangle = v_{T_3}, \quad \langle U \rangle = v_U,
\end{aligned} \tag{29}$$

where the complex phases are all explicitly displayed. Note that there are only three phases, ϕ_S , ϕ_u and ϕ_d in the VEVs, along with a purely imaginary VEV of T_2 . To proceed we introduce:

$$Y_+^{u,d} = \frac{1}{\sqrt{2}}(\Phi_1^{u,d} + \Phi_2^{u,d}), \quad Y_-^{u,d} = \frac{i}{\sqrt{2}}(\Phi_1^{u,d} - \Phi_2^{u,d}), \tag{30}$$

whose VEVs are given by $\langle Y_+^{u,d} \rangle = \sqrt{2}v_1^{u,d} \cos \phi^{u,d}$, $\langle Y_-^{u,d} \rangle = \sqrt{2}v_1^{u,d} \sin \phi^{u,d}$. Then we redefine them as

$$\begin{pmatrix} \Phi_L^u \\ \Phi_H^u \\ \Phi_-^u \end{pmatrix} = \begin{pmatrix} s_{\gamma^u} c_{\phi^u} & s_{\gamma^u} s_{\phi^u} & c_{\gamma^u} \\ c_{\gamma^u} c_{\phi^u} & c_{\gamma^u} s_{\phi^u} & -s_{\gamma^u} \\ -s_{\phi^u} & c_{\phi^u} & 0 \end{pmatrix} \begin{pmatrix} Y_+^u \\ Y_-^u \\ \Phi_3^u \end{pmatrix}, \tag{31}$$

where

$$c_{\gamma^u} = \cos \gamma^u = \sqrt{2}v_3^u/v_u, \quad s_{\gamma^u} = \sin \gamma^u = 2v_1^u/v_u, \tag{32}$$

$$c_{\phi^u} = \sin \phi^u, \quad s_{\phi^u} = \sin \phi^u, \quad v_u = \sqrt{2(v_3^u)^2 + 4(v_1^u)^2}, \tag{33}$$

and similarly for the down sector. As we see from Eqs. (30)-(32), only Φ_L^u and Φ_L^d have a nonvanishing VEV. According to (10), the new Higgs doublet supermultiplets are defined as

$$\Phi_I^u = (h_I^u, \tilde{h}_I^u) = \begin{pmatrix} h_I^{u+} & \tilde{h}_I^{u+} \\ h_I^{u0} & \tilde{h}_I^{u0} \end{pmatrix}, \quad \Phi_I^d = (h_I^d, \tilde{h}_I^d) = \begin{pmatrix} h_I^{d0} & \tilde{h}_I^{d0} \\ h_I^{d-} & \tilde{h}_I^{d-} \end{pmatrix} \tag{34}$$

with $I = L, H, -$. As we see from Eq. (32), h_L^{u0} and h_L^{d0} are assumed to have a nonvanishing VEV: $\langle h_L^{u0,d0} \rangle = v_{u,d}/\sqrt{2}$. The light and heavy MSSM-like Higgs scalars are then given by

$$\frac{1}{\sqrt{2}}(H + iA) = (h_L^{d0})^* s_\beta - (h_L^{u0}) c_\beta, \quad \frac{1}{\sqrt{2}}(v + h + iG) = (h_L^{d0})^* c_\beta + (h_L^{u0}) s_\beta, \quad (35)$$

$$G^+ = -(h_L^{d-})^* c_\beta + (h_L^{u+}) s_\beta, \quad H^+ = (h_L^{d-})^* s_\beta + h_L^u c_\beta, \quad (36)$$

$$v = \sqrt{v_u^2 + v_d^2} (\simeq 246 \text{ GeV}), \quad \tan \beta = v_u/v_d, \quad (37)$$

where $c_\beta = \cos \beta$, $s_\beta = \sin \beta$, G and G^+ are the NG fields, and we assumed that the mass of H and A is much larger than M_Z . As in the case of the MSSM, the couplings of $\Phi_L^{u,d}$ are flavor-diagonal, while the extra heavy neutral fields $h_-^{u0,d0} = (\varphi_-^{u,d} + i\chi_-^{u,d})/\sqrt{2}$, $h_H^{u0,d0} = (\varphi_H^{u,d} + i\chi_H^{u,d})/\sqrt{2}$ can have flavor-changing couplings.

The SM like Higgs boson h defined in (35) is the lightest Higgs boson in this model. The tree-level upper bound of its mass has due to the cubic superpotential (25) an extra contribution in addition to the MSSM one:

$$m_h^2 \lesssim M_Z^2 c_{2\beta}^2 + \frac{1}{2} v^2 s_{2\beta}^2 \left[c_{\gamma^u}^2 s_{\gamma^d}^2 (\lambda_1'')^2 + \frac{1}{2} c_{2\gamma^u} s_{2\gamma^d} \cos(\phi^u - \phi^d) \lambda_1'' \lambda_2'' + s_{\gamma^u}^2 [c_{\gamma^d}^2 (\lambda_2'')^2 + s_{\gamma^d}^2 \cos(\phi^u + \phi^d) (\lambda_3'')^2] \right], \quad (38)$$

where c_{γ^u} , etc. are defined in (32). So with the radiative correction included [50] one can bring m_h easily to ~ 125 GeV, around which an excess of events is reported at LHC [51, 52].

By integrating out the SM singlet supermultiplets, we can obtain the effective theory. The Higgs superpotential of the effective theory can be written as

$$W_{II}^{\text{eff}} = \mu_{11}(\Phi_1^u \Phi_1^d + \Phi_2^u \Phi_2^d) + \mu_{32} \Phi_3^u \Phi_2^d + \mu_{32}^* \Phi_3^u \Phi_1^d + \mu_{13} \Phi_1^u \Phi_3^d + \mu_{13}^* \Phi_2^u \Phi_3^d, \quad (39)$$

where $\mu_{11} = \mu_{22}$ is real. W_{II}^{eff} yields a \mathcal{P}_{II} invariant scalar potential, but breaks Q_6 flavor symmetry softly. The soft breaking is consistent with Q_6 according to (**Th1**). In terms of the redefined Higgs superfields (31), W_{II}^{eff} becomes

$$W_{II}^{\text{eff}} = \mu_L \Phi_L^u \Phi_L^d + \mu_{LH} \Phi_L^u \Phi_H^d + \mu_{HL} \Phi_H^u \Phi_L^d + \mu_H \Phi_H^u \Phi_H^d + \mu_- \Phi_-^u \Phi_-^d + \mu_{-L} \Phi_-^u \Phi_L^d + \mu_{L-} \Phi_L^u \Phi_-^d + \mu_{-H} \Phi_-^u \Phi_H^d + \mu_{H-} \Phi_H^u \Phi_-^d, \quad (40)$$

and similarly the SSB Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_{II\text{soft}}^{\text{eff}} = & m_{uL}^2 |h_L^u|^2 + [m_{uLH}^2 h_L^{u*} h_H^u + h.c.] + m_{uH}^2 |h_H^u|^2 + m_{u-}^2 |h_-^u|^2 \\ & + m_{dL}^2 |h_L^d|^2 + [m_{dLH}^2 h_L^{d*} h_H^d + h.c.] + m_{dH}^2 |h_H^d|^2 + m_{d-}^2 |h_-^d|^2 \\ & + [B_L h_L^u h_L^d + B_{LH} h_L^u h_H^d + B_{HL} h_H^u h_L^d + B_H h_H^u h_H^d + B_- h_-^u h_-^d + h.c.] \\ & + [B_{-L} h_-^u h_L^d + B_{L-} h_L^u h_-^d + B_{-H} h_-^u h_H^d + B_{H-} h_H^u h_-^d + h.c.] . \end{aligned} \quad (41)$$

Note that the parameters appearing in Eqs. (40) and (41) are all real. This is because all the fields can be redefined without a non-trivial phase rotation in such way that their VEVs become real. That is, CP is restored in the Higgs sector. Because of the restored CP invariance in the Higgs sector, the CP even and odd fields do not mix at the tree level.

Accordingly, the mass matrix for the neutral CP even Higgs bosons assumes the form

$$\mathbf{M}_{\text{NHeven}}^2 = \begin{pmatrix} c_{2\beta}^2 M_z^2 & -c_{2\beta} s_{2\beta} M_z^2 & 0 & 0 & 0 & 0 \\ -c_{2\beta} s_{2\beta} M_z^2 & \frac{2B_L}{S_{2\beta}} + s_{2\beta}^2 M_z^2 & -\frac{\hat{m}_{uLH}^2}{c_\beta} & \frac{\hat{m}_{dLH}^2}{s_\beta} & \frac{\mu_1^2}{c_\beta} & -\frac{\mu_2^2}{s_\beta} \\ 0 & -\frac{\hat{m}_{uLH}^2}{c_\beta} & -\hat{m}_{uH}^2 & -B_H & \mu_3^2 & -B_{H-} \\ 0 & \frac{\hat{m}_{dLH}^2}{s_\beta} & -B_H & -\hat{m}_{dH}^2 & -B_{-H} & \mu_4^2 \\ 0 & \frac{\mu_1^2}{c_\beta} & \mu_3^2 & -B_{-H} & -\hat{m}_{u-}^2 & -B_- \\ 0 & -\frac{\mu_2^2}{s_\beta} & -B_{H-} & \mu_4^2 & -B_- & -\hat{m}_{d-}^2 \end{pmatrix} \quad (42)$$

in the basis $(h, H, \varphi_H^u, \varphi_H^d, \varphi_-^u, \varphi_-^d)$, where

$$\mu_1^2 = \mu_{L-}\mu_- + \mu_{LH}\mu_{-H} + \mu_L\mu_{-L}, \quad \mu_2^2 = \mu_{HL}\mu_{H-} + \mu_L\mu_{L-} + \mu_- \mu_{-L}, \quad (43)$$

$$-\hat{m}_{uLH}^2 = -m_{uLH}^2 + \mu_{HL}\mu_L + \mu_H\mu_{LH} + \mu_{H-}\mu_{L-}, \quad (44)$$

$$-\hat{m}_{dLH}^2 = -m_{dLH}^2 + \mu_H\mu_{HL} + \mu_L\mu_{L-} + \mu_{-H}\mu_{-L}, \quad (45)$$

$$\mu_3^2 = \mu_{H-}\mu_- + \mu_H\mu_{-H} + \mu_{HL}\mu_{-L}, \quad \mu_4^2 = \mu_H\mu_{H-} + \mu_{LH}\mu_{L-} + \mu_- \mu_{-H}, \quad (46)$$

$$-\hat{m}_{u(d)H}^2 = -m_{u(d)H}^2 + \mu_H^2 + \mu_{HL(LH)}^2 + \mu_{H-(-H)}^2 - (+)\frac{1}{2}c_{2\beta}M_z^2, \\ -\hat{m}_{u(d)-}^2 = -m_{u(d)-}^2 + \mu_-^2 + \mu_{-H(H-)}^2 + \mu_{-L(L-)}^2 - (+)\frac{1}{2}c_{2\beta}M_z^2. \quad (46)$$

The mass matrix for the CP odd neutral Higgs bosons is found to be

$$\mathbf{M}_{\text{NHodd}}^2 = \begin{pmatrix} \frac{B_L}{c_\beta s_\beta} & -\frac{\hat{m}_{uLH}^2}{c_\beta} & \frac{\hat{m}_{dLH}^2}{s_\beta} & \frac{\mu_1^2}{c_\beta} & \frac{\mu_2^2}{s_\beta} \\ -\frac{\hat{m}_{uLH}^2}{c_\beta} & -\hat{m}_{uH}^2 + c_{\theta_W}^2 c_{2\beta} M_Z^2 & B_H & \mu_3^2 & B_{H-} \\ \frac{\hat{m}_{dLH}^2}{s_\beta} & B_H & -\hat{m}_{dH}^2 - c_{\theta_W}^2 c_{2\beta} M_Z^2 & B_{-H} & \mu_4^2 \\ \frac{\mu_1^2}{c_\beta} & \mu_3^2 & B_{-H} & -\hat{m}_{u-}^2 + c_{\theta_W}^2 c_{2\beta} M_Z^2 & B_- \\ \frac{\mu_2^2}{s_\beta} & B_{H-} & \mu_4^2 & B_- & -\hat{m}_{d-}^2 - c_{\theta_W}^2 c_{2\beta} M_Z^2 \end{pmatrix} \quad (47)$$

in the basis $(A, \chi_H^{u0}, \chi_H^{d0}, \chi_-^{u0}, \chi_-^{d0})$. Similarly, we obtain the mass matrix of the charged Higgs bosons:

$$\mathbf{M}_C^2 = \begin{pmatrix} \frac{B_L}{c_\beta s_\beta} & -\frac{\hat{m}_{uLH}^2}{c_\beta} & -\frac{\hat{m}_{dLH}^2}{s_\beta} & \frac{\mu_1^2}{c_\beta} & \frac{\mu_2^2}{s_\beta} \\ -\frac{\hat{m}_{uLH}^2}{c_\beta} & -\hat{m}_{uH}^2 & B_H & \mu_3^2 & B_{H-} \\ -\frac{\hat{m}_{dLH}^2}{s_\beta} & B_H & -\hat{m}_{dH}^2 & B_{-H} & \mu_4^2 \\ \frac{\mu_1^2}{c_\beta} & \mu_3^2 & B_{-H} & -\hat{m}_{u-}^2 & B_- \\ \frac{\mu_2^2}{s_\beta} & B_{H-} & \mu_4^2 & B_- & -\hat{m}_{d-}^2 \end{pmatrix} \quad (48)$$

in the basis $(H^+, h_H^{u+}, h_H^{d-*}, h_-^{u+}, h_-^{d-*})$.

B. The Yukawa sector

The most general, renormalizable, $Q_6 \times Z_4 \times R$ invariant superpotential W_Y in the Yukawa sector is given by ³ $W_Y = W_Q + W_L$, where W_Q and W_L are given in (7) with the Yukawa

³ The $Q_6 \times Z_4 \times R$ assignment is given in Table 1.

matrices

$$\mathbf{Y}^{u1(d1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_b^{u(d)} \\ 0 & Y_{b'}^{u(d)} & 0 \end{pmatrix}, \quad \mathbf{Y}^{u2(d2)} = \begin{pmatrix} 0 & 0 & Y_b^{u(d)} \\ 0 & 0 & 0 \\ -Y_{b'}^{u(d)} & 0 & 0 \end{pmatrix}, \quad \mathbf{Y}^{u3(d3)} = \begin{pmatrix} 0 & Y_c^{u(d)} & 0 \\ Y_c^{u(d)} & 0 & 0 \\ 0 & 0 & Y_a^{u(d)} \end{pmatrix}, \quad (49)$$

$$\mathbf{Y}^{e1} = \begin{pmatrix} -Y_c^e & 0 & Y_b^e \\ 0 & Y_c^e & 0 \\ Y_{b'}^e & 0 & 0 \end{pmatrix}, \quad \mathbf{Y}^{e2} = \begin{pmatrix} 0 & Y_c^e & 0 \\ Y_c^e & 0 & Y_b^e \\ 0 & Y_{b'}^e & 0 \end{pmatrix}, \quad \mathbf{Y}^{e3} = 0, \quad (50)$$

$$\mathbf{Y}^{\nu1} = \begin{pmatrix} -Y_c^\nu & 0 & 0 \\ 0 & Y_c^\nu & 0 \\ Y_{b'}^\nu & 0 & 0 \end{pmatrix}, \quad \mathbf{Y}^{\nu2} = \begin{pmatrix} 0 & Y_c^\nu & 0 \\ Y_c^\nu & 0 & 0 \\ 0 & Y_{b'}^\nu & 0 \end{pmatrix}, \quad \mathbf{Y}^{\nu3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_a^\nu \end{pmatrix}. \quad (51)$$

All the parameters appearing above are real, because we assume that CP is spontaneously broken. From the Yukawa interactions along with the Yukawa matrices (49)-(51) and the VEV structure of the \mathcal{P}_{II} invariant case (given in (29)), we obtain the fermion mass matrices. As we will see that the resulting mass matrix of the quarks is of a nearest neighbor interaction (NNI) type which is one of the successful Ansätze for the quark mass matrix [53, 54]⁴. The Q_6 assignment of the fermions is so chosen that the NNI type of the mass matrix occurs from flavor symmetry (see also [56]). The Majorana mass term for N^c is given by $W_M = \frac{1}{2}M_1(N_1^c N_1^c + N_2^c N_2^c) + \lambda_M \langle T_3 \rangle N_3^c N_3^c$. We first discuss the quark sector and then the lepton sector in the following sections.

V. FERMION MASS AND MIXING

A. The quark sector and the Cabibbo-Kobayashi-Maskawa (CKM) mixing

In the background \mathcal{P}_{II} , the fermion mass matrices $\mathbf{m}_{u,d}$ take the form

$$\mathbf{m}_{u,d} = \begin{pmatrix} 0 & C_{u,d} & \frac{B_{u,d}}{\sqrt{2}} e^{-i\phi^{u,d}} \\ C_{u,d} & 0 & \frac{B'_{u,d}}{\sqrt{2}} e^{i\phi^{u,d}} \\ -\frac{B'_{u,d}}{\sqrt{2}} e^{-i\phi^{u,d}} & \frac{B'_{u,d}}{\sqrt{2}} e^{i\phi^{u,d}} & A_{u,d} \end{pmatrix}, \quad (52)$$

where $A_{u,d} = Y_a^{u,d} v_3^{u,d}$, $B_{u,d} = \sqrt{2} Y_b^{u,d} v_1^{u,d}$, $B'_{u,d} = \sqrt{2} Y_{b'}^{u,d} v_1^{u,d}$, $C_{u,d} = Y_c^{u,d} v_3^{u,d}$. The phases in the matrix of Eq. (52) can be factorized as $\mathbf{m}_{u,d}^r = P_{u,d} \mathbf{m}_{u,d} P_{u,d}$, where

$$P_{u,d} = \text{diag.}\{e^{i\phi^{u,d}}, e^{-i\phi^{u,d}}, 1\}. \quad (53)$$

Then we do a 45 degrees rotation in the (1-2) plane to bring $\mathbf{m}_{u,d}^r$ into

$$\hat{\mathbf{m}}_{u,d} = R_L^T \mathbf{m}_{u,d}^r R_R = \begin{pmatrix} 0 & C_{u,d} & 0 \\ -C_{u,d} & 0 & B_{u,d} \\ 0 & B'_{u,d} & A_{u,d} \end{pmatrix}, \quad (54)$$

⁴ See [55] for a review for different Ansätze.

generating a non-trivial quark mixing matrix given by

$$K = R_L^T P_u^\dagger P_d R_L = \begin{pmatrix} \cos \phi & i \sin \phi & 0 \\ i \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \phi = \phi^u - \phi^d, \quad (55)$$

where

$$R_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad R_R = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}. \quad (56)$$

The CKM mixing matrix is then obtained as $V_{\text{CKM}} = O_u^T K O_d$, where $O_{u,d}$ diagonalize the matrices of Eq. (54). An excellent fit is obtained with the following choice of parameters at $\mu = 1$ TeV:

$$\begin{aligned} A_u/m_t &= 7.031 \times 10^{-3}, B_u/m_t = -3.124 \times 10^{-3}, B'_u/m_t = 1.000, C_u/m_t = 1.750 \times 10^{-3}, \\ A_d/m_b &= 0.9200, B_d/m_b = 0.04406, B'_d/m_b = 0.3899, C_d/m_b = 4.203 \times 10^{-3}, \\ \phi &= 0.08375. \end{aligned} \quad (57)$$

The resulting quark masses at $\mu = 1$ TeV are:

$$m_u = 0.910 \text{ MeV}, \quad m_c = 542 \text{ MeV}, \quad m_d = 2.21 \text{ MeV}, \quad m_s = 44.5 \text{ MeV}, \quad (58)$$

where we have used $m_t = 151.3$ GeV and $m_b = 2.46$ GeV. These values are to be compared with quark masses extrapolated from low energy scale to $\mu = 1$ TeV [57]:

$$\begin{aligned} m_u &= 0.85 \sim 1.55 \text{ MeV}, \quad m_d = 2.05 \sim 2.85 \text{ MeV}, \\ m_s &= 39.6 \sim 64.4 \text{ MeV}, \quad m_c = 502 \sim 570 \text{ MeV}, \\ m_b &= 2.39 \sim 2.53 \text{ GeV}, \quad m_t = 150.3 \sim 151.8 \text{ GeV}, \end{aligned} \quad (59)$$

where we have updated the result of [57] by using the updated quark masses given in PDG 2011 [58], while neglecting the uncertainties due to the RG running. The input values of Eq. (57) give the output for the CKM parameters:

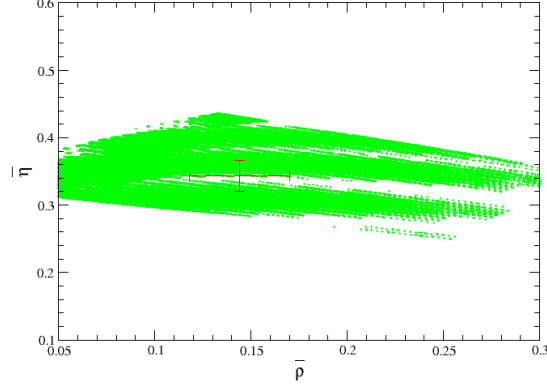
$$\begin{aligned} \lambda &= 0.2260, \quad A = 0.792, \quad \bar{\rho} = 0.148, \quad \bar{\eta} = 0.331, \\ \sin 2\beta &= 0.675, \quad \alpha = 92.8 [\text{deg}], \quad \beta = 21.2 [\text{deg}], \quad \gamma = 65.9 [\text{deg}], \end{aligned} \quad (60)$$

which should be compared with the fit result of the CKMfitter group [59, 60]

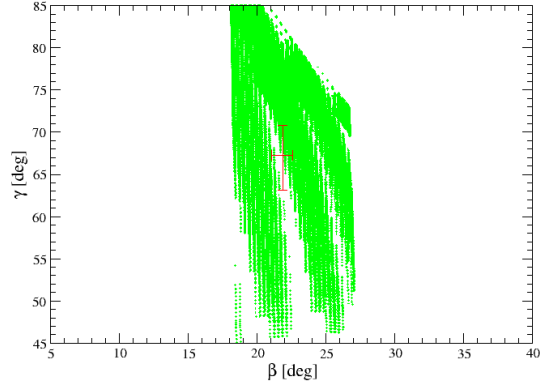
$$\lambda = 0.22539^{+0.00062}_{-0.00095}, \quad A = 0.801^{+0.026}_{-0.014}, \quad (61)$$

$$\begin{aligned} \bar{\rho} &= 0.144^{+0.023}_{-0.026}, \quad \bar{\eta} = 0.343^{+0.015}_{-0.014}, \quad \sin 2\beta = 0.691^{+0.020}_{-0.020}, \\ \alpha &= 90.9^{+3.5}_{-4.1} [\text{deg}], \quad \beta = 21.84^{+0.80}_{-0.76} [\text{deg}], \quad \gamma = 67.3^{+4.2}_{-3.5} [\text{deg}]. \end{aligned} \quad (62)$$

Fig. 1 shows the prediction in the $\bar{\rho} - \bar{\eta}$ plane (a) and in the $\beta - \gamma$ plane (b) for \mathcal{P}_{II} . The CKMfitter group best-fit value (62) is indicated in these plots. We see from Eqs. (58) and Fig. 1 that the model \mathcal{P}_{II} reproduces the quark masses, CKM mixings and the CP violating phase in an excellent way.



a)



b)

FIG. 1: The predictions in the $\bar{\rho} - \bar{\eta}$ plane (a) and $\beta - \gamma$ plane (b) for the \mathcal{P}_{II} invariant model, where we have used as the input parameters; the quark masses, λ and A given in Eqs. (59) and (61), respectively. We also have imposed the constraints on the quark masses [58]: $2m_s/(m_u + m_d) = 22 \sim 30$, $m_s/m_d = 17 \sim 22$, $m_u/m_d = 0.35 \sim 0.60$, $(1/2)(m_u + m_d)(2\text{GeV}) = (3.0 \sim 4.8)$ MeV. The crosses denote one σ value given in [59].

B. Lepton sector

From the VEV structure of the \mathcal{P}_{II} invariant case, given in (29), we see that the Majorana mass term for N_3^c is real, because $\langle T_3 \rangle$ is real: $M_N = (M_1, M_1, M_3 = a_{\nu^c} v_{T_3})$. The Dirac neutrino and charged lepton mass matrices are:

$$\mathbf{m}_{\nu D} = \begin{pmatrix} -C_\nu e^{-i\phi^u} & C_\nu e^{i\phi^u} & 0 \\ C_\nu e^{i\phi^u} & C_\nu e^{-i\phi^u} & 0 \\ B'_\nu e^{-i\phi^u} & B'_\nu e^{i\phi^u} & A_\nu \end{pmatrix}, \quad \mathbf{m}_\ell = \begin{pmatrix} -C_\ell e^{i\phi^d} & C_\ell e^{-i\phi^d} & B_\ell e^{i\phi^d} \\ C_\ell e^{-i\phi^d} & C_\ell e^{i\phi^d} & B_\ell e^{-i\phi^d} \\ B'_\ell e^{i\phi^d} & B'_\ell e^{-i\phi^d} & 0 \end{pmatrix}, \quad (63)$$

which (by the seesaw formula) lead to the light neutrino Majorana mass matrix

$$M'_\nu = m_0 \begin{pmatrix} 2\rho_2^2 \cos(2\phi^u) & 0 & 2i\rho_2\rho_4 \sin(2\phi^u) \\ 0 & 2\rho_2^2 \cos(2\phi^u) & 2\rho_2\rho_4 \\ 2i\rho_2\rho_4 \sin(2\phi^u) & 2\rho_2\rho_4 & -\rho_3^2 + 2\rho_4^2 \cos(2\phi^u) \end{pmatrix}, \quad (64)$$

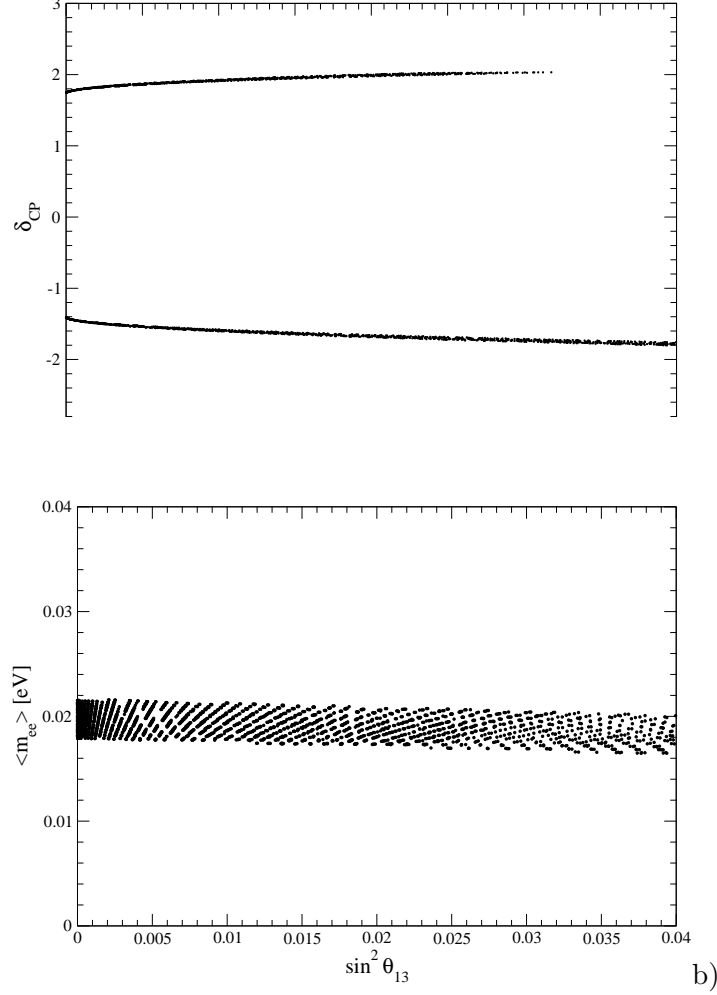


FIG. 2: The prediction in the $\sin^2 \theta_{13} - \delta_{CP}$ plane (a) and in the $\sin^2 \theta_{13} - \langle m_{ee} \rangle$ plane (b) for \mathcal{P}_{II} . We have used $\sin^2 \theta_{12}$, Δm_{21}^2 and Δm_{23}^2 given in (65) as the input values of the parameters, and $\phi_d = \phi_u - 0.08375$.

where $\rho_2^2 = (C_\nu)^2/M_1$, $\rho_3^2 = -(A_\nu)^2/M_3$, $\rho_4^2 = (B'_\nu)^2/M_1$. We have assumed that M_1 is positive, while M_3 is negative. We make the matrix (64) real by redefining $\nu_1 = i\nu'_1$. The resulting mass matrix can be diagonalized by an orthogonal matrix. As for the charged lepton mass matrix \mathbf{m}_ℓ , we can obtain hierarchical masses, e.g., $m_e \sim B'_\ell$, $m_\mu \sim C_\ell$, $m_\tau \sim B_\ell$. Since the relative phase $\phi = \phi^u - \phi^d$ is fixed in the quark sector, there are seven independent parameters in the lepton sector. When $\phi^u = 0$, the neutrino mass matrix above yields only a tiny $\sin^2 \theta_{13} \simeq m_e/m_\mu \sim 10^{-3}$ [5], which is too small [61–65]. For non-zero ϕ^u , we obtain $\sin \theta_{13} \propto \sin 2\phi^u$, which can be small or large. We use the three charged lepton masses, $\sin^2 \theta_{12}$, Δm_{21}^2 and Δm_{23}^2 , and vary the value of ϕ^u while ϕ^d is set equal to $\phi^u - 0.08375$ (see (57)), where the up-dated neutrino oscillation parameters [61] in which the recent results of T2K [62] and MINOS [63] experiments are included are:

$$\begin{aligned} \Delta m_{21}^2 &= 7.59^{+0.20}_{-0.18} \times 10^{-5} \text{ eV}^2, \quad \Delta m_{23}^2 = 2.4^{+0.08}_{-0.09} \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.312^{+0.017}_{-0.015}, \quad \sin^2 \theta_{13} = 0.016^{+0.008}_{-0.006}, \quad \sin^2 \theta_{23} = 0.52 \pm 0.06. \end{aligned} \quad (65)$$

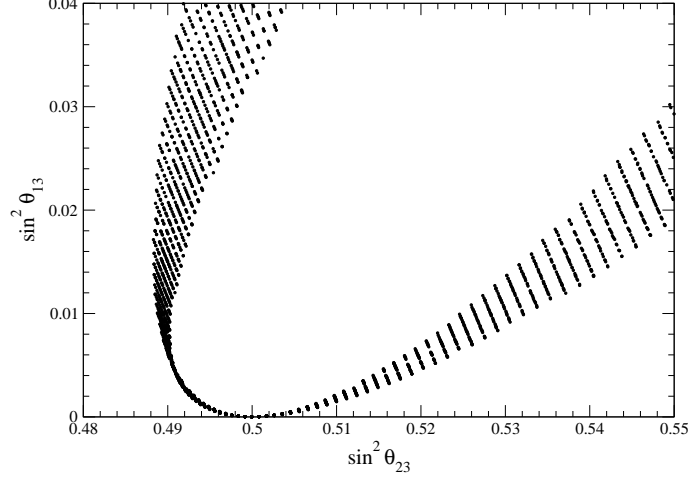


FIG. 3: $\sin^2 \theta_{13}$ against $\sin^2 \theta_{23}$ in the case of \mathcal{P}_{II} for the same input parameters as Fig. 2.

We below display the predictions of the model in different two dimensional planes. Fig. 2 (a) shows the Dirac phase δ_{CP} (in the convention of Ref. [58]) against $\sin^2 \theta_{13}$. We see that the model predicts nearly maximal CP violation. It is also possible to predict the effective neutrino mass $\langle m_{ee} \rangle$ as a function of $\sin^2 \theta_{13}$. If the mixing is tri-bimaximal [66], for instance, then $m_{\nu_2} \simeq \sqrt{\Delta m_{23}^2} 3\sqrt{3}/2\sqrt{2} \simeq 0.09$ eV. In Fig. 2 (b) we plot the prediction in the $\sin^2 \theta_{13} - \langle m_{ee} \rangle$ plane. The deviation from the maximal mixing has terms proportional to m_e/m_μ and to $\sin^2 2\phi_u$. In Fig. 3 we plot $\sin^2 \theta_{13}$ against $\sin^2 \theta_{23}$. Once the $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ are measured, the model prediction can be tested by experiments (see [67] and [68]).

VI. SUPPRESSING FCNCs BY FLAVOR SYMMETRY AND CP VIOLATIONS BY SELF-ALIGNMENT OF PHASES

A. Tree-level FCNCs and the Cheng-Sher mechanism

Before we go to FCNCs coming from the supersymmetry breaking, we shall demonstrate that the Cheng-Sher mechanism [23] to suppress the tree-level FCNCs is partially embedded in the Q_6 model (see also [56]). Consider the mass difference of the neutral Kaons $\Delta M_K \simeq 3.5 \times 10^{-15}$ GeV, for instance, and assume that there is a neutral Higgs boson H with mass M_H which has a flavor changing Yukawa coupling with the down and strange quarks: $(Y_{ds}\bar{d}_L s_R + Y_{sd}\bar{s}_L d_R)H + h.c.$. Then the contribution to ΔM_K can be estimated as [34]

$$\Delta M_K = 2 | \langle \bar{K}^0 | \bar{s}_R d_L \bar{s}_L d_R | K^0 \rangle | \left(\frac{|Y_{ds} Y_{sd}^*|}{M_H^2} \right) = \frac{1}{2} f_K^2 B'_K M_K \left(\frac{M_K}{m_s + m_d} \right)^2 \left(\frac{|Y_{ds} Y_{sd}^*|}{M_H^2} \right), \quad (66)$$

where $f_K \simeq 0.16$ GeV is the Kaon decay constant, B'_K is a bag parameter for the matrix element in (66). Using $M_K \simeq 0.5$ GeV, $m_s \simeq 0.1$ GeV and $m_d \simeq 5 \times 10^{-3}$ GeV, we obtain

$$\Delta M_K \simeq 0.56 \left(\frac{|Y_{ds} Y_{sd}^*|}{M_H^2} \right) \text{ GeV}, \quad (67)$$

where we have assumed that $B'_K = 1$. From (67) we obtain a lower bound for M_H : $M_H \gtrsim 1.3 \times 10^4 \sqrt{|Y_{ds} Y_{sd}^*|} \text{ TeV}$. Therefore, if $|Y_{ds}|, |Y_{sd}| \simeq \sqrt{m_s m_d}/v \sim 10^{-4}$ ($v = 246$ GeV) is

satisfied, then the lower bound reduces to $\sim O(1)$ TeV. Cheng and Sher showed that this is true in more general cases: If the Yukawa couplings satisfy $(\mathbf{Y})_{ij} \sim \sqrt{m_i m_j}/v$, then the lower bound of the mass of flavor changing neutral Higgs bosons is $O(\text{few})$ TeV.

We will show that in the Q_6 model this mechanism is partially embedded. To this end, we consider the Yukawa matrices (49) and find:

$$\mathbf{Y}^{u1} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.05\sqrt{m_c m_t} \\ 0 & 17\sqrt{m_c m_t} & 0 \end{pmatrix} \frac{\sqrt{2}}{v s_{\gamma^u} s_\beta}, \quad \mathbf{Y}^{u2} \simeq \begin{pmatrix} 0 & 0 & 1.3\sqrt{m_u m_t} \\ 0 & 0 & 0 \\ -408\sqrt{m_u m_t} & 0 & 0 \end{pmatrix} \frac{\sqrt{2}}{v s_{\gamma^u} s_\beta},$$

$$\mathbf{Y}^{u3} \simeq \begin{pmatrix} 0 & 12\sqrt{m_u m_c} & 0 \\ 12\sqrt{m_u m_c} & 0 & 0 \\ 0 & 0 & 0.007\sqrt{m_t m_t} \end{pmatrix} \frac{\sqrt{2}}{v c_{\gamma^u} s_\beta}, \quad (68)$$

$$\mathbf{Y}^{d1} \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.3\sqrt{m_s m_b} \\ 0 & 2.9\sqrt{m_s m_b} & 0 \end{pmatrix} \frac{\sqrt{2}}{v s_{\gamma^d} c_\beta}, \quad \mathbf{Y}^{d2} \simeq \begin{pmatrix} 0 & 0 & 1.5\sqrt{m_d m_b} \\ 0 & 0 & 0 \\ -13\sqrt{m_d m_b} & 0 & 0 \end{pmatrix} \frac{\sqrt{2}}{v s_{\gamma^d} c_\beta},$$

$$\mathbf{Y}^{d3} \simeq \begin{pmatrix} 0 & 1.0\sqrt{m_d m_s} & 0 \\ 1.0\sqrt{m_d m_s} & 0 & 0 \\ 0 & 0 & 0.9\sqrt{m_b m_b} \end{pmatrix} \frac{\sqrt{2}}{v c_{\gamma^d} c_\beta}, \quad (69)$$

where we have used the central values of the parameters given in (57). The large (3, 2) and (3, 1) elements of \mathbf{Y}^{u1} and \mathbf{Y}^{u2} , respectively, can induce flavor changing decay of the top quark, $t \rightarrow c + \bar{q} + q$ and $t \rightarrow u + \bar{q} + q$. However, the rate is very small. The large (1, 2) and (2, 1) elements of \mathbf{Y}^{u3} contribute dominantly to the mass difference of the neutral D meson $\Delta M_D \simeq 1.4 \times 10^{-14}$ GeV. From a similar estimate that we have done above we find

$$M_H \gtrsim \frac{\sqrt{2 \cdot 12 \cdot 12 m_u m_c}}{v c_{\gamma^u} s_\beta} \times 2.1 \times 10^3 \text{ TeV} \simeq \frac{8.3}{c_{\gamma^u} s_\beta} \text{ TeV}. \quad (70)$$

Here we have demonstrated that the Cheng-Sher mechanism is partially working in the Q_6 model. In the lepton sector one can do the same calculations, but it turned out that the constraints on the lower bound on M_H are much weaker than (70) (see for instance [69]).

B. FCNCs and CP violations from the soft supersymmetry breaking (SSB)

We work in the super CKM basis and use mass insertion parameters $(\delta_{ij}^\alpha)_{LL,RR,LR}$ ($\alpha = u, d, e$) to parameterize FCNCs and CP violations coming from the SSB sector. We start with the mass eigenstates of the quarks $u_{iL} \rightarrow u'_{iL} = (U_{uL})_{ij} u_{jL}$, $u'_{iR} \rightarrow u^c_{iR} = (U_{uR})_{ij}^* u^c_{jR}$, where $u'_1 = u, u'_2 = c, u'_3 = t$, and similarly for the other matter fermions. Then we go to the super CKM basis for the matter fermions: $\tilde{u}_{iL} \rightarrow \tilde{u}^s_{iL} = (U_{uL})_{ij} \tilde{u}_{jL}$, $\tilde{u}_{iR} \rightarrow \tilde{u}^s_{iR} = (U_{uR})_{ij} \tilde{u}_{jR}$, etc. In the super CKM basis, the squark mass matrices become

$$\tilde{\mathbf{M}}_u^{s2} = \begin{pmatrix} m_{\tilde{u}_i L} m_{\tilde{u}_j L} [\mathbf{1}_{ij} + (\delta_{ij}^u)_{LL}] & (\Delta_{ii}^u)_{LR} \mathbf{1}_{il} + m_{\tilde{u}_i L} m_{\tilde{u}_l R} (\delta_{il}^u)_{LR} \\ (\Delta_{kk}^u)_{LR}^* \mathbf{1}_{kj} + m_{\tilde{u}_k L} m_{\tilde{u}_j R} (\delta_{jk}^{*u})_{LR} & m_{\tilde{u}_k R} m_{\tilde{u}_l R} [\mathbf{1}_{kl} + (\delta_{kl}^u)_{RR}] \end{pmatrix}, \quad (71)$$

and

$$\tilde{\mathbf{M}}_d^{s2} = \begin{pmatrix} m_{\tilde{d}_i L} m_{\tilde{d}_j L} [\mathbf{1}_{ij} + (\delta_{ij}^d)_{LL}] & (\Delta_{ii}^d)_{LR} \mathbf{1}_{il} + m_{\tilde{d}_i L} m_{\tilde{d}_l R} (\delta_{il}^d)_{LR} \\ (\Delta_{kk}^d)_{LR}^* \mathbf{1}_{kj} + m_{\tilde{d}_k L} m_{\tilde{d}_j R} (\delta_{jk}^{*d})_{LR} & m_{\tilde{d}_k R} m_{\tilde{d}_l R} [\mathbf{1}_{kl} + (\delta_{kl}^d)_{RR}] \end{pmatrix}, \quad (72)$$

respectively, where [32]

$$m_{u_i} \mathbf{1}_{ij} = (U_{uL}^\dagger \mathbf{m}_u U_{uR})_{ij} , \quad m_{d_i} \mathbf{1}_{ij} = (U_{dL}^\dagger \mathbf{m}_d U_{dR})_{ij} , \quad (73)$$

$$m_{\tilde{u}_i L(R)}^2 = (U_{uL(R)}^\dagger \tilde{\mathbf{m}}_{Q(u)}^2 U_{uL(R)})_{ii} + m_{u_i}^2 + c_{L(R)}^u M_Z^2 ,$$

$$m_{\tilde{d}_i L(R)}^2 = (U_{dL(R)}^\dagger \tilde{\mathbf{m}}_{Q(d)}^2 U_{dL(R)})_{ii} + m_{d_i}^2 + c_{L(R)}^d M_Z^2 ,$$

$$(\delta_{ij}^{u(d)})_{LL} = [(U_{u(d)L}^\dagger \tilde{\mathbf{m}}_Q^2 U_{u(d)L})_{ij} - (U_{u(d)L}^\dagger \tilde{\mathbf{m}}_Q^2 U_{u(d)L})_{ii} \mathbf{1}_{ij}] / m_{\tilde{u}_i(\tilde{d}_i)L} m_{\tilde{u}_j(\tilde{d}_j)L} , \quad (74)$$

$$(\delta_{ij}^{u(d)})_{RR} = [(U_{u(d)R}^\dagger \tilde{\mathbf{m}}_{u(d)}^2 U_{u(d)R})_{ij} - (U_{u(d)R}^\dagger \tilde{\mathbf{m}}_{u(d)}^2 U_{u(d)R})_{ii} \mathbf{1}_{ij}] / m_{\tilde{u}_i(\tilde{d}_i)R} m_{\tilde{u}_j(\tilde{d}_j)R} ,$$

$$(\Delta_{ij}^u)_{LR} = (U_{uL}^\dagger)_{ik} \{ -\mu_{IJ} (Y_{kl}^{uI})^* < h_J^{d0} > + A_{kl}^{*u}(\mathbf{m}_u)_{kl} \} (U_{uR})_{lj} ,$$

$$(\Delta_{ij}^d)_{LR} = (U_{dL}^\dagger)_{ik} \{ \mu_{JI} (Y_{kl}^{dI}) < h_J^{u0} > + A_{kl}^{*d}(\mathbf{m}_d)_{kl} \} (U_{dR})_{lj} , \quad (75)$$

$$(\delta_{ij}^{u(d)})_{LR} = [(\Delta_{ij}^{u(d)})_{LR} - (\Delta_{ii}^{u(d)})_{LR} \mathbf{1}_{ij}] / m_{\tilde{u}_i(\tilde{d}_i)L} m_{\tilde{u}_j(\tilde{d}_j)R} . \quad (76)$$

So far no assumption on flavor symmetry is made, and the squark mass matrices (71) and (72) cover the case of more than one pairs of Higgs doublet supermultiplets.

If three generations of a family are put into a one-dimensional and two-dimensional irreps of any group, then the soft scalar mass matrix for the sfermions has always a diagonal form:

$$\tilde{\mathbf{m}}_{Q,L}^2 = m_{\tilde{q},\tilde{\ell}}^2 \text{diag.} (a_L^{q,\ell}, a_L^{q,\ell}, b_L^{q,\ell}) , \quad \tilde{\mathbf{m}}_\alpha^2 = m_{\tilde{q},\tilde{\ell}}^2 \text{diag.} (a_R^\alpha, a_R^\alpha, b_R^\alpha) \quad (\alpha = u, d, e) , \quad (77)$$

where $m_{\tilde{q},\tilde{\ell}}$ denote the average of the squark and slepton masses, respectively, and $(a_{L(R)}, b_{L(R)})$ are dimensionless free real parameters of $O(1)$. Further, flavor symmetry imposes the soft trilinear coupling matrices (A terms) to have the same structure as the Yukawa coupling matrices. They are real, because we impose CP invariance at the Lagrangian level. The imaginary parts of δ_{ij}^α and Δ_{ij}^α contribute to CP violating processes. Recall that the soft scalar mass matrices are real, because they are diagonal. The unitary matrices U_{uL} , etc. are complex, and so $(\delta_{ij}^\alpha)_{LL,RR}$ are complex in general. Note that the unitary matrices have the form $U = RPO$ in the case of \mathcal{P}_I and $U = PRO$ in the case of \mathcal{P}_{II} (see (53) and (56)), respectively, where only the phase matrices P are complex. Since all the soft scalar mass matrices are diagonal with the structure $\tilde{\mathbf{m}}^2 \sim \text{diag.}(a, a, b)$, they commute with P and R , so that $(\delta_{ij}^\alpha)_{LL,RR}$ have no imaginary part. Further, the phase of the A term contribution to $(\Delta_{ij}^\alpha)_{LR}$ in (75) comes from the complex VEVs (see (29)), because CP is only spontaneously broken. Therefore, the A term contribution has the same phase structure as the corresponding fermion mass matrix \mathbf{m}_α . That is, defining the A parameters as $h_\gamma^\alpha = A_\gamma^\alpha Y_\gamma^\alpha$ ($\alpha = u, d, e$, $\gamma = a, b, b', c$), where h_γ^α are the soft tri-linear couplings, and the Yukawa couplings are defined in (49)-(51), we find that, except for $(\Delta_{ij}^e)_{LR}$ in the case of \mathcal{P}_{II} , the A term contributions to $(\Delta_{ij}^\alpha)_{LR}$ are real, too. Consequently, there is no CP violation originating from the SSB sector. Only the μ term contributions to $(\Delta_{ij}^\alpha)_{LR}$ are complex. The stringent constraints coming from the EDMs [34, 70] are automatically satisfied in this way of phase alignment, except for the μ term contributions. To suppress the contribution to the EDMs, therefore, one should have relatively small μ 's [44]. This is general if there are more than one pair of Higgs doublet supermultiplets.

The theoretical values of $(\delta_{ij}^\alpha)_{LL,RR,LR}$ for the present model are calculated below ⁵, where

$$\Delta a_L^{q,\ell} = a_L^{q,\ell} - b_L^{q,\ell}, \quad \Delta a_R^\alpha = a_R^\alpha - b_R^\alpha, \quad \tilde{A}_\gamma^\alpha = A_\gamma^\alpha / m_{\tilde{q},\tilde{\ell}} \quad (\alpha = u, d, e, \quad \gamma = a, b, b', c) , \quad (78)$$

⁵ We do not consider the RG running effects of the SSB parameters [71]. See also [72].

are introduced, and we assume that $m_{\tilde{q}_i L, R} m_{\tilde{q}_i L, R} = \delta_{ij} m_{\tilde{q}}^2$, $m_{\tilde{\ell}_i L, R} m_{\tilde{\ell}_i L, R} = \delta_{ij} m_{\tilde{\ell}}^2$. In Table 2, theoretical values of δ 's and their experimental bounds [34, 35] are summarized.

$$\begin{aligned} \text{Im}(\delta_{11}^d)_{LR} &\simeq 10^{-4} \text{Im} \left[(-0.20 c_{\phi^d} + i 1.0 s_{\phi^d}) \tilde{\mu}_+^d - (0.20 s_{\phi^d} + i 2.2 c_{\phi^d}) \tilde{\mu}_-^d + 0.05 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1} \\ (\delta_{12}^d)_{LR} &\simeq 10^{-5} \left[1.7 \left(\tilde{A}_a^d - \tilde{A}_b^d - \tilde{A}_{b'}^d + \tilde{A}_c^d \right) \right. \\ &\quad \left. - (8.6 c_{\phi^d} + i 1.1 s_{\phi^d}) \tilde{\mu}_+^d - (8.6 s_{\phi^d} - i 1.1 c_{\phi^d}) \tilde{\mu}_-^d + 1.0 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1}, \end{aligned} \quad (80)$$

$$\begin{aligned} (\delta_{21}^d)_{LR} &\simeq 10^{-4} \left[-0.16 \left(\tilde{A}_a^d - \tilde{A}_{b'}^d \right) + 0.19 \left(\tilde{A}_b^d - \tilde{A}_c^d \right) \right. \\ &\quad \left. + (0.78 c_{\phi^d} - i 3.8 s_{\phi^d}) \tilde{\mu}_+^d + (0.78 s_{\phi^d} + i 3.8 c_{\phi^d}) \tilde{\mu}_-^d - 1.1 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1}, \\ (\delta_{13}^d)_{LR} &\simeq 10^{-5} \left[-4.2 \left(\tilde{A}_a^d - \tilde{A}_b^d \right) \right. \\ &\quad \left. - (4.4 c_{\phi^d} + i 2.8 s_{\phi^d}) \tilde{\mu}_+^d - (4.4 s_{\phi^d} - i 2.8 c_{\phi^d}) \tilde{\mu}_-^d + 0.42 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1}, \end{aligned} \quad (81)$$

$$\begin{aligned} (\delta_{31}^d)_{LR} &\simeq 10^{-3} \left[-0.40 \left(\tilde{A}_a^d - \tilde{A}_{b'}^d \right) \right. \\ &\quad \left. + (2.1 c_{\phi^d} - i 9.6 s_{\phi^d}) \tilde{\mu}_+^d + (2.1 s_{\phi^d} + i 9.6 c_{\phi^d}) \tilde{\mu}_-^d - 0.004 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1}, \\ (\delta_{23}^d)_{LR} &\simeq 10^{-4} \left[1.6 \tilde{A}_a^d - 1.9 \tilde{A}_b^d + 0.3 \tilde{A}_{b'}^d \right. \\ &\quad \left. + (1.8 c_{\phi^d} - i 0.06 s_{\phi^d}) \tilde{\mu}_+^d + (1.8 s_{\phi^d} + i 0.06 c_{\phi^d}) \tilde{\mu}_-^d + 0.03 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1}, \end{aligned} \quad (82)$$

$$\begin{aligned} (\delta_{32}^d)_{LR} &\simeq 10^{-3} \left[-1.7 \left(\tilde{A}_a^d - \tilde{A}_b^d \right) \right. \\ &\quad \left. + (8.8 c_{\phi^d} + i 2.2 s_{\phi^d}) \tilde{\mu}_+^d + (8.8 s_{\phi^d} - i 2.2 c_{\phi^d}) \tilde{\mu}_-^d + 0.02 \tilde{\mu}_3^d \right] \tilde{m}_{\tilde{q}}^{-1}, \end{aligned}$$

$$\text{Im}(\delta_{11}^u)_{LR} \simeq 3.5 \times 10^{-4} \text{Im}(\tilde{\mu}_3^u) \tilde{m}_{\tilde{q}}^{-1}, \quad (83)$$

$$\begin{aligned} (\delta_{12}^u)_{LR} &\simeq 10^{-3} \left[-0.03 \left(\tilde{A}_a^u - \tilde{A}_{b'}^u \right) + 0.03 \left(\tilde{A}_b^u - \tilde{A}_c^u \right) \right. \\ &\quad \left. - (0.01 c_{\phi^u} - i 2.2 s_{\phi^u}) \tilde{\mu}_+^u - (0.01 s_{\phi^u} + i 2.2 c_{\phi^u}) \tilde{\mu}_-^u + 0.6 \tilde{\mu}_3^u \right] \tilde{m}_{\tilde{q}}^{-1}, \end{aligned} \quad (84)$$

$$\begin{aligned} (\delta_{21}^u)_{LR} &\simeq 10^{-3} \left[-0.03 \left(\tilde{A}_a^u - \tilde{A}_{b'}^u \right) + 0.03 \left(\tilde{A}_b^u - \tilde{A}_c^u \right) \right. \\ &\quad \left. + (0.92 c_{\phi^u} + i 0.01 s_{\phi^u}) \tilde{\mu}_+^u + (0.92 s_{\phi^u} - i 0.01 c_{\phi^u}) \tilde{\mu}_-^u - 1.6 \tilde{\mu}_3^u \right] \tilde{m}_{\tilde{q}}^{-1}, \end{aligned}$$

where

$$\begin{aligned} \tilde{\mu}_+^u &= - \left(\frac{\mu_{11}}{m_{\tilde{q}}} \frac{c_{\phi^d} s_{\gamma^d}}{s_{\gamma^u}} + \sqrt{2} \frac{\text{Re}(\mu_{13})}{m_{\tilde{q}}} \frac{c_{\gamma^d}}{s_{\gamma^u}} \right) \tan^{-1} \beta, \\ \tilde{\mu}_-^u &= \left(\frac{\mu_{11}}{m_{\tilde{q}}} \frac{s_{\phi^d} s_{\gamma^d}}{s_{\gamma^u}} - \sqrt{2} \frac{\text{Im}(\mu_{13})}{m_{\tilde{q}}} \frac{c_{\gamma^d}}{s_{\gamma^u}} \right) \tan^{-1} \beta, \\ \tilde{\mu}_3^u &= -2 \frac{\text{Re}(\mu_{31} e^{-i\phi^d})}{m_{\tilde{q}}} \frac{s_{\gamma^d}}{s_{\gamma^u}} \tan^{-1} \beta, \quad \tilde{\mu}_3^d = 2 \frac{\text{Re}(\mu_{13} e^{-i\phi^u})}{m_{\tilde{q}}} \frac{s_{\gamma^u}}{s_{\gamma^d}} \tan \beta, \\ \tilde{\mu}_+^d &= \left(\frac{\mu_{11}}{m_{\tilde{q}}} \frac{c_{\phi^u} s_{\gamma^u}}{s_{\gamma^d}} + \sqrt{2} \frac{\text{Re}(\mu_{31})}{m_{\tilde{q}}} \frac{c_{\gamma^u}}{s_{\gamma^d}} \right) \tan \beta, \quad \tilde{\mu}_-^d = \left(-\frac{\mu_{11}}{m_{\tilde{q}}} \frac{s_{\phi^u} s_{\gamma^u}}{s_{\gamma^d}} + \sqrt{2} \frac{\text{Im}(\mu_{31})}{m_{\tilde{q}}} \frac{c_{\gamma^u}}{s_{\gamma^d}} \right) \tan \beta, \end{aligned} \quad (85)$$

and the angles s_{γ^d} , etc. are defined in (32).. The μ term contributions above are complex and so contribute to EDMs. We see from (79) and (83) that the strong constraint $|\text{Im}(\delta_{11}^{u,d})'_{LR}| \lesssim 10^{-6}$ from the neutron EDM [34, 70] can be satisfied if μ 's are relatively small compared with the soft scalar masses [44, 45]. In contrast to this, $(\delta_{23}^d)_{LR}$ and $(\delta_{32}^d)_{LR}$,

which contribute to $b \rightarrow s + \gamma$, are well below the upper limit $10^{-2} (500 \text{ GeV}/m_{\tilde{q}})^2$. Further, the imaginary part of $(\delta_{23}^d)_{LR}$ and $(\delta_{32}^d)_{LR}$ contributes to the CP violation in the $B_s^0 - \bar{B}_s^0$ mixing. Their size is small compared with $(\delta_{23}^d)_{LL,RR}$ given in Table 2, which however do not contribute to CP violation because they are real at the tree-level. At the one-loop level they can be complex, and the size can be as large as the tree-level size under certain circumstances [45]. With this (fine-tuned) one-loop contribution the theoretical value of the dimuon asymmetry can become close [45] to the order of the D0 measurement [73]. This large CP violation has been toned down by CDF [74] and then by the recent measurement at LHCb [75] (see also [60]). Therefore, that fine tuning is no longer necessary. Another interesting observation can be made on flavor violating decays of the top quark, i.e. $t \rightarrow u(c) + g$. From $(\delta_{13,31,23,32}^u)_{LR}$, we find the ratio $B(t \rightarrow u + g)/B(t \rightarrow c + g) \sim 10^{-4}$, which is consistent with the bound recently found by ATLAS [78].

	Exp. bound	Q_6 Model (\mathcal{P}_{II})
$\sqrt{ \text{Re}(\delta_{12}^d)_{LL,RR}^2 }$	$4.0 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$(LL) 4.0 \times 10^{-4} \Delta a_L^q, (RR) 3.3 \times 10^{-2} \Delta a_R^d$
$\sqrt{ \text{Re}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} }$	$2.8 \times 10^{-3} \tilde{m}_{\tilde{q}}$	$3.6 \times 10^{-3} \sqrt{\Delta a_L^q \Delta a_R^d}$
$\sqrt{ \text{Re}(\delta_{12}^d)_{LR}^2 }$	$4.4 \times 10^{-3} \tilde{m}_{\tilde{q}}$	Eq. (80)
$\sqrt{ \text{Re}(\delta_{13}^d)_{LL,RR}^2 }$	$9.8 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$(LL) 1.0 \times 10^{-2} \Delta a_L^q, (RR) 8.0 \times 10^{-2} \Delta a_R^d$
$\sqrt{ \text{Re}(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR} }$	$1.8 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$2.8 \times 10^{-2} \sqrt{\Delta a_L^q \Delta a_R^d}$
$\sqrt{ \text{Re}(\delta_{13}^d)_{LR}^2 }$	$3.3 \times 10^{-2} \tilde{m}_{\tilde{q}}$	Eq. (81)
$\sqrt{ \text{Re}(\delta_{12}^u)_{LL,RR}^2 }$	$1.5 \times 10^{-1} \tilde{m}_{\tilde{q}}$	$(LL) 2.9 \times 10^{-8} \Delta a_L^q, (RR) 4.3 \times 10^{-1} \Delta a_R^u$
$\sqrt{ \text{Re}(\delta_{12}^u)_{LL}(\delta_{12}^u)_{RR} }$	$1.7 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$1.1 \times 10^{-4} \sqrt{\Delta a_L^q \Delta a_R^u}$
$\sqrt{ \text{Re}(\delta_{12}^u)_{LR}^2 }$	$3.1 \times 10^{-2} \tilde{m}_{\tilde{q}}$	Eq. (84)
$ (\delta_{23}^d)_{LL,RR} $	$8.2 \tilde{m}_{\tilde{q}}^2$	$(LL) 3.9 \times 10^{-2} \Delta a_L^q, (RR) 3.5 \times 10^{-1} \Delta a_R^d$
$ (\delta_{23}^d)_{LR} $	$1.6 \times 10^{-2} \tilde{m}_{\tilde{q}}^2$	Eq. (82)
$ \text{Im}(\delta_{11}^d)_{LR} $	$3.0 \times 10^{-6} \tilde{m}_{\tilde{q}}$	Eq. (79)
$ \text{Im}(\delta_{11}^u)_{LR} $	$5.9 \times 10^{-6} \tilde{m}_{\tilde{q}}$	Eq. (83)

TABLE II: Experimental bounds on $(\delta_{ij}^{u,d})_{LL,RR,LR}$ [34, 35] and their theoretical values in the quark sector of the Q_6 model for \mathcal{P}_{II} (see also Kobayashi *et al.* in [40]), where $\tilde{m}_{\tilde{q}}$ denotes $m_{\tilde{q}}/500 \text{ GeV}$, and $\Delta a_{L,R}$ and \tilde{A} are given in (78).

Now we come to the lepton sector. As it is noticed, the phase alignment does not work for the lepton sector in the case of \mathcal{P}_{II} . There are two phases, ϕ^u and ϕ^d , that enter into the CKM and MNS matrices. One combination $\phi = \phi^u - \phi^d$ is basically fixed around 0.0838 (see (57)) to produce the correct Kobayashi-Maskawa CP phase. The other would be fixed if CP violation in the neutrino sector is precisely measured. Then the phase that enters into the soft mass insertions would be completely fixed. In other words we can express the mass insertions as a function of $\sin^2 \theta_{13}$, because in the case of \mathcal{P}_{II} the CP phase δ_{CP} and $\sin^2 \theta_{13}$ are intimately related. This is done in Fig. 4. We plot $|(\delta_{12}^e)_{LL}|/\Delta a_L^\ell$ and $|(\delta_{12}^e)_{RR}|/\Delta a_R^\ell$ against $\sin^2 \theta_{13}$, respectively. We see from Fig. 4 that $|(\delta_{12}^e)_{LL}|/\Delta a_L^\ell$ does not change very much as $\sin^2 \theta_{13}$ varies, while $|(\delta_{12}^e)_{RR}|/\Delta a_R^\ell$ depends significantly on $\sin^2 \theta_{13}$.

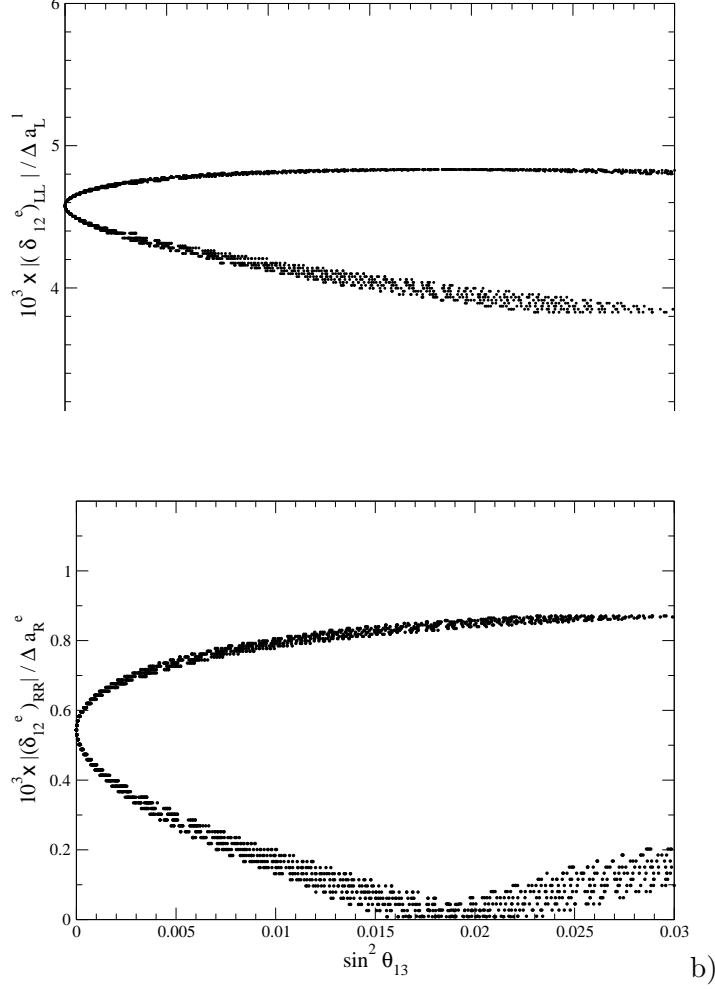


FIG. 4: $|(\delta_{12}^e)_{LL}|/\Delta a_L^e$ against $\sin^2 \theta_{13}$ (a) and $|(\delta_{12}^e)_{RR}|/\Delta a_R^e$ (b). The parameter space is the same as for Fig. 2.

Unfortunately, there is no common range of $\sin^2 \theta_{13}$ such that the both insertions become small. The experimental bounds are [34, 35]⁶:

$$|(\delta_{12}^e)_{LL}| \lesssim 1.8 \times 10^{-4} \tilde{m}_{\tilde{\ell}}^2, \quad |(\delta_{12}^e)_{RR}| \lesssim 3.6 \times 10^{-4} \tilde{m}_{\tilde{\ell}}^2, \quad |(\delta_{12}^e)_{LR}| \lesssim 3.3 \times 10^{-6} \tilde{m}_{\tilde{\ell}}^2, \quad (86)$$

where $\tilde{m}_{\tilde{\ell}} = m_{\tilde{\ell}}/300\text{GeV}$. In Fig. 5 we plot $|(\bar{\delta}_{12}^e)_{LR}|$ against $\sin \phi^d$ with ϕ fixed at 0.08375, where we varied the dimensionless parameters \tilde{A}_γ^e (defined in (78)) from -5 to 5 , and $|(\bar{\delta}_{12}^e)_{LR}|$ is defined as

$$(\bar{\delta}_{12}^e)_{LR} = (\delta_{12}^e)_{LR} \tilde{m}_{\tilde{\ell}}^{-1} / \left(\sum_{\gamma=a,b,b',c} |\tilde{A}_\gamma^e| \right). \quad (87)$$

We see from Fig. 5 that the size of $(\delta_{12}^e)_{LR}$ is of order of the experimental bound (86), and that it has no strong dependence of $\sin \phi^d$ and hence of $\sin^2 \theta_{13}$.

⁶ The recent experimental bounds $B(\mu \rightarrow e + \gamma) \lesssim 2.4 \times 10^{-12}$ [76] and $B(\tau \rightarrow \mu(e) + \gamma) \lesssim 4.4(3.3) \times 10^{-8}$ [77] are included.

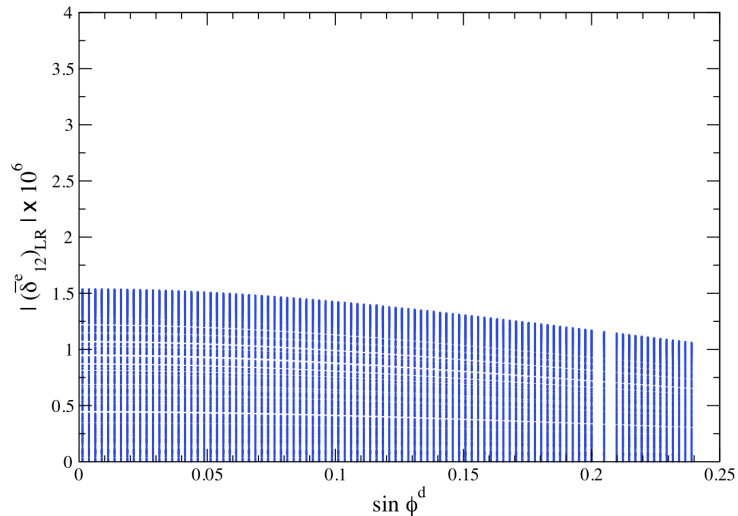


FIG. 5: $(\bar{\delta}_{12}^e)_{LR}$ against $\sin \phi^d$, where the dimensionless parameters \tilde{A}_γ^e are varied from -5 to 5 . $(\bar{\delta}_{ij}^e)_{LR}$ is defined in (87).

We see thus from Tables 2 and Figs. 4 and 5 that Q_6 flavor symmetry can well soften the FCNC problem of the SSB sector. The recent LHC data suggest that the superpartner masses are heavier than those assumed here [79, 80]. If this is the case the constraints are better satisfied.

VII. SUMMARY

Obviously, the flavor problem is one of the most difficult problems. So far we do not have a magic bullet to solve this problem. Non-abelian flavor symmetry can soften the problem and therefore might give a hint to a solution. In this contribution I restricted myself to renormalizable models of flavor with a non-abelian discrete flavor symmetry, which is at most softly broken at some tera scale. This setting without supersymmetry may lead to problems because of FCNCs and of a fine tuning in the scalar potential. The fine tuning problem may be softened in supersymmetric models. In softly broken supersymmetric theories, the soft breaking terms of flavor symmetry can be classified, which I discussed in a very general form.

To point out basic features of supersymmetric models of flavor with multiple Higgs doublets I considered a specific model based on the finite flavor group Q_6 [7]. To make spontaneous breaking of CP possible, the Higgs sector was further extended [46] so as to include a certain set of SM singlet Higgs multiplets. With them the upper bound of the lightest Higgs mass upper bound can slightly increase. In this SM singlet Higgs sector, flavor symmetry and CP are spontaneously broken at a tera scale, which is very close to supersymmetry breaking scale. This may give a hint that flavor symmetry, CP and supersymmetry are broken within the same sector.

The Q_6 model of flavor yields interesting predictions for the CKM and MNS mixing

parameters, which compare very well with experimental data. Large B-factories [81, 82] with more precise determination of quark masses [83] will be able to test the predictions in the quark sector [84]. In the lepton sector, the predictions for the ground state \mathcal{P}_{II} are particularly interesting, because the neutrino mass hierarchy is inverted with nearly maximal CP violation along with nearly maximal mixing of atmospheric neutrinos: The CP phase, the average neutrino mass $\langle m_{ee} \rangle$, and $\sin^2 \theta_{23}$ can be expressed as a function of $\sin^2 \theta_{13}$. These predictions can also be tested by the future neutrino experiments (see [67] and ([68]).

Non-abelian flavor symmetry can not only relate the fermion masses and mixing parameters, but also suppress FCNCs. Flavor symmetry embeds the Cheng-Sher mechanism automatically, which suppresses the tree-level FCNCs, making the flavor symmetry scale decrease down to tera scale, and suppress FCNCs coming from the soft breaking of supersymmetry as well. The CP problem of softly broken supersymmetric theories is solved by virtue of spontaneous CP violation. This implies that the soft supersymmetry breaking parameters such as the gluino mass are all real. The phase of the trilinear soft breaking A terms, since they arise spontaneously, will align with the phases in the fermion mass matrices. A self-alignment mechanism is going on. Thus the A terms do not generate CP violation. Thanks to flavor symmetry, the mass insertion parameters $(\delta_{ij}^\alpha)_{LL,RR,LR}$ depend on only few independent parameters. Their ratio, e.g., $(\delta_{12}^e)_{LL}/(\delta_{13}^e)_{RR}$, is fixed, which is a unique prediction. Therefore, flavor violations are controlled by flavor symmetry. Thus new observations of flavor and CP violations can also test flavor symmetry at tera scale [85].

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